Matrix model and Holographic Baryons in the D0-D4 background

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Abstract

We study on the spectrum and short-distance two-body force of holographic baryons by the matrix model, which is derived from Sakai-Sugimoto model in D0-D4 background (D0-D4/D8 system). The matrix model is derived by using the standard technique in string theory and it can describe multi-baryon system. We re-derive the action of the matrix model from open string theory on the wrapped baryon vertex, which is embedded in the D0- D4/D8 system. The matrix model offers a more systematic approach to the dynamics of the baryons at short distances. In our system, we find that the matrix model describe stable baryonic states only if $\zeta = U_{Q_0}^3/U_{KK}^3 < 2$, where $U_{Q_0}^3$ is related to the number density of smeared D0-branes. This result in our paper is exactly the same as some previous presented results studied on this system as [27]. We also compute the baryon spectrum ($k = 1$ case) and short-distance two-body force of baryons ($k = 2$ case). The baryon spectrum is modified and could be able to fit the experimental data if we choose suitable value for ζ. And the short-distance two-body force of baryons is also modified by the appearance of smeared D0-branes from the original Sakai-Sugimoto model. If $\zeta > 2$, we find that the baryon spectrum would be totally complex and an attractive force will appear in the short-distance interaction of baryons, which may consistently correspond to the existence of unstable baryonic states.

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1 Introduction

QCD as the underlying fundamental theory to physicists, has achieved great successes in particle physics and nuclear physics, however we still can not describe or predict the behavior of baryons or nucleons exactly. In high energy physics, it is well-known that nuclear physics remains one of the most difficult and intriguing branches. The key problem is that we have not yet understood much about the strong-coupling QCD since to study on the strong-coupling QCD is hopeless by using the techniques from perturbative quantum field theory directly. However about two decades ago, the discovery of AdS/CFT and gauge/string duality [1, 2, 3, 4] became one of the turning points. The AdS/CFT correspondence has been recognized as a promising framework to understand non-perturbative aspects of gauge field theory, thus it may be able to provide a new way to study on nuclear physics.

There have been many applications or models of AdS/CFT for studying on strong-coupling QCD such as [5, 6, 7, 8, 9]. Several top-down models for QCD inspired by AdS/CFT have also been proposed in recent years. The most well-known D4-D8/$\overline{\text{D8}}$ brane system based on Witten's work [10] was built up by Sakai and Sugimoto [11, 12] (Sakai-Sugimoto model). The Sakai-Sugimoto model consists of N_c D4-branes and N_f D8-branes as probes. The D4-branes are compactified on a cycle, offering color numbers, which represent pure Yang-Mills in the low energy effective theory. Supersymmetry is broken down by introducing the anti-periodic boundary conditions for fermions on the compactified cycle. Since only the low energy theory is concerned in this model, so the dynamics of background geometry produced by N_c D4-branes, can be described by Type II A supergravity on the large-N_c limit. The N_f species of massless flavored quarks are introduced by the embedding N_f pairs of probe D8/$\overline{\text{D8}}$-branes. The flavor D8/$\overline{\text{D8}}$-branes are connected at the IR region of the D4 solitonic solution, which corresponds to the geometrically broken chiral symmetry in the confined phase for the dual field theory. In this geometry, the low energy effective theory of light meson sector comes from the worldvolume theory on the connected D8/$\overline{\text{D8}}$-branes. There is an other solution for the background geometry for this model, i.e. the D4 black brane solution, which corresponds to the deconfined phase for the dual field theory [13] (However, in fact it is less clear that what phase the deconfined geometry corresponds to in the dual field theory [14]). As it is known that baryons are D-branes wrapped on non-trivial cycles [15, 16, 17], so in this model, baryons are identified as D4-branes wrapped on a four-cycle which is named as the "baryon vertex". And it has turned out baryons can be treated as a small instanton configuration in the worldvolume gauge theory on the probe D8-branes [18]. According to these viewpoints, there are many researches on baryons or nuclear matters by holography, such as the phase structure [16, 19, 20, 21] and the interaction [22, 23, 24]. However some results from the Sakai-Sugimoto model is still not realistic to QCD. One of the most likely reasons may be that the Sakai-Sugimoto model is a theory with large N_c limit, but the real QCD is not. Therefore some generalizations or modifications of this model have been proposed such as [25], and the backreaction of the flavor branes has also been considered recently as [26]. In our paper, we follow [25] and use the gauge/gravity duality to study the dynamics of multi-baryons from D0-D4/D8 system by matrix model proposed in [24]. By using the matrix model, we calculate some basic properties of holographic baryons: the spectrum of baryons and the effective two-body potential both from Sakai-Sugimoto in D0-D4 background as an extension of [25, 27].

The original matrix model is the effective theory for D0-branes [28], which can also be understood as a dual description of D=11 supergravity. As a generalization, the matrix model in [24], is proposed as the effective theory for baryons or nucleons by holography. In this matrix model, the rank of the matrix represents the number of baryons and k baryon branes produce $U(k)$ symmetry for k-body baryons. The diagonal elements of matrices represent the positions of k baryons while the off-diagonal elements are integrated out. Besides, the size of baryons are related to the classical values of a pair of complex $k \times N_f$ rectangular matrices, which describe the dynamics of the strings connecting the flavor branes and the baryon vertices

in low energy effective theory. With all together, it comes to the well-known Atiyah-Drinfeld-Hitchin-Manin (ADHM) matrix of instantons. So in our paper, we re-derive the matrix model from Sakai-Sugimoto in D0-D4 background by using standard technique in string theory. Such background geometry is produced by N_c D4-branes with N_0 smeared D0-branes. In this background, we follow the original idea in [15, 17], and recall that baryons can be identified as D4-branes wrapped on the 4-cycle. Here we start to use D4'-branes to distinguish such a baryon vertex from those D4-branes who are responsible for the background geometry. In the large N_c limit, the dynamics of the open strings with both ends on the D4'-branes and which connects the D4'-branes and the D8-branes are also relevant, thus the dynamics of k baryons are described by $U(k)$ gauge theory. And the theory on the D4'-branes is reduced to a 0+1 dimensional matrix model by considering only the zero modes along the S^4 on which the D4'- and the D8-branes are wrapped. Then it is clear that the matrix model is just the low energy effective theory for the baryon vertices.

Our motivation for this paper is to study the holographic baryons by the matrix model derived from Sakai-Sugimoto model in D0-D4 background (i.e. D0-D4/D8 system). In this paper, since the appearance of smeared D0-branes could be able to modify the results about baryons from the original Sakai-Sugimoto model, thus our results may be more close to the realistic physics. On the other hand, accommodating many bodies of baryons by such a matrix model would be very easy, so we can even use this model to describe the interaction among multi-baryons. The outline of this paper is as follows. In Section II, we give a brief review of Sakai-Sugimoto model in D0-D4 background (D0-D4/D8 system). In Section III, we start to derive the matrix model in the D0-D4 background by considering D4'-D8 gauge theory compactified on S^4. Since we keep k baryons close to each other, so only the short-distance effects are relevant. It turns out that our matrix model can describe stable baryonic states only for $\zeta = U_{Q_0}^3/U_{KK}^3 < 2$, where $U_{Q_0}^3$ is related to the number density of smeared D0-branes. And the constraint for stable baryonic states ($\zeta < 2$) is exactly the same as [27]. And as two simple examples, we calculate the energy functions of static configurations with $k = 1$ and $k = 2$, for one and two flavor(s). In Section IV, we use our matrix model derived from D0-D4/D8 system to study baryon spectrum. We determine the size of the holographic baryons with the case of $k = 1$, also for one and two flavor(s). Again it turns out that the baryon spectrum does make sense only for $\zeta < 2$, otherwise baryons may not exist or be unstable. In Section V, we study the case of $k = 2$ and use the instantons of ADHM matrix as data to calculate a baryon-baryon potential at short distance for two-flavor case. By integrating out the auxiliary gauge potential in 0+1 dimension, it also turns out that there is a universal repulsive core of the two-body force, but modified by the appearance of smeared D0-branes. And a short-distance attractive force would appear if $\zeta > 2$, which consistently corresponds to the existence of unstable baryonic states in two-body system. The summary and conclusion are given in the final section.

2 A brief review of Sakai-Sugimoto model in the D0-D4 background

Here we begin to use D4'-brane to distinguish baryon vertex from those N_c D4-branes which are producing the background geometry. As we are going to derive the low energy effective theory for D4'-branes from Sakai-Sugimoto model in D0-D4 background, thus in this section, we first review the Sakai-Sugimoto model in D0-D4 background geometry briefly. Some of the results in this section are already presented in [25, 27].

2.1 D0-D4 background geometry

By taking the near horizon limit, the solution of D4-branes with smeared D0 charges in Type IIA supergravity reads [25]

$$\begin{aligned} ds^2 &= \left(\frac{U}{R}\right)^{3/2}\left(H_0^{1/2}\eta_{\mu\nu}dx^\mu dx^\nu + H_0^{-1/2}f(U)\,d\tau^2\right) \\ &\quad + H_0^{1/2}\left(\frac{R}{U}\right)^{3/2}\left(\frac{1}{f(U)}dU^2 + U^2 d\Omega_4^2\right). \end{aligned} \tag{1}$$

We have written this metric (1) in string frame and τ is a periodic variable. With the near horizon limit we also have the formulas for dilaton, Ramond-Ramond (R-R) 4-form and the 2-form which is

$$e^\phi = g_s\left(\frac{U}{R}\right)^{3/4} H_0^{3/4};\ f_2 = \frac{A}{U^4}\frac{1}{H_0^2}dU\wedge d\tau;\ f_4 = dC_3 = B\epsilon_4; \tag{2}$$

where we have

$$A = \frac{(2\pi l_s)^7 g_s N_0}{\omega_4 V_4};\ B = \frac{(2\pi l_s)^3 N_c g_s}{\omega_4};\ H_0 = 1 + \frac{U_{Q_0}^3}{U^3};\ f(U) = 1 - \frac{U_{KK}^3}{U^3}, \tag{3}$$

$d\Omega_4$, ϵ_4 and $\omega_4 = 8\pi^2/3$ are the line element, the volume form and the volume of a unit S^4. U_{KK} is the coordinate radius of the bottom of the bubble, and V_4 is the volume of D4-brane. N_0 and N_c are the numbers of D0- and D4-branes respectively. D0-branes are smeared in the $x^0, ..., x^3$ directions. The relations to the QCD variables are deformed as

$$R^3 = \frac{\lambda l_s^2}{2M_{KK}};\ g_s = \frac{\lambda}{2\pi M_{KK} N_c l_s};\ U_{KK} = \frac{2}{9}M_{KK}\lambda l_s^2 H_0(U_{KK}). \tag{4}$$

Here in order to keep the back reaction of D0-brane, we also required N_0 to be of order N_c as in [29].

2.2 Embedded D8-branes and baryon vertex in D0-D4 background

In the Sakai-Sugimoto model, flavors of the dual gauge field theory are introduced by N_f D8/$\overline{\text{D8}}$-branes as probes embedded in the background geometry. Here we embed these N_f D8/$\overline{\text{D8}}$-branes into the background described by (1) as [11, 25]. By taking the probe limit i.e. $N_c, N_0 \gg N_f$ which makes the back reaction of the D8/$\overline{\text{D8}}$-brane to the background negligible. The fermions created by open strings can be added to the fundamental representation of $U(N_c)\times U_{R/L}(N_f)$ which is treated as groups of chiral symmetry, while the gauge fields are in adjoint representation.

We employ the viewpoint in Sakai-Sugimoto model, baryons have been provided as a D4'-brane wrapped on S^4 which is called the baryon vertex [15, 16, 17]. Such D4'-branes have to attach the ends of N_c fundamental strings since the S^4 is supported by N_c units of a R-R flux in the supergravity solution. In this way, the baryon charge equals to N_c quark charge in the corresponding field theory. According to these arguments, baryons are also the D4'-branes wrapped on S^4 in our D0-D4 background geometry (1). As we are going to study on the baryons, thus we will focus on the baryon vertex in the Sakai-Sugimoto model in D0-D4 background in next sections.

3 The matrix model from D0-D4/D8 system

In this section, we derive the matrix quantum mechanics and we are going to use the action to study the baryon spectrum and the two-body interaction by an effective potential. As mentioned, the matrix model is

just the low energy effective action on the k D4'-branes embedded in flavor N_f D8-branes in the geometry described by (1). Here we first give our result i.e. the action of our matrix model in D0-D4/D8 system, and we leave the details for derivation in next two subsections.

3.1 The action

The action of our matrix model for baryons from D0-D4/D8 system is

$$\begin{aligned} S \;=\; & \frac{\lambda N_c M_{KK}}{54\pi}(1+\zeta)^{3/2}\,\mathrm{Tr}\int dt\Big[\left(D_0X^M\right)^2-\frac{2}{3}\left(1-\frac{1}{2}\zeta\right)M_{KK}^2\left(X^4\right)^2 \\ & +D_0\bar{\omega}_i^{\dot{\alpha}}D_0\omega_{i\dot{\alpha}}-\frac{1}{6}\left(1-\frac{1}{2}\zeta\right)M_{KK}^2\bar{\omega}_i^{\dot{\alpha}}\omega_{i\dot{\alpha}} \\ & +\frac{3^6\pi^2}{4\lambda^2M_{KK}^4}\frac{1}{(1+\zeta)^4}\left(\vec{D}\right)^2+\vec{D}\cdot\vec{\tau}^{\dot{\alpha}}_{\dot{\beta}}\bar{X}^{\dot{\beta}\alpha}X_{\dot{\alpha}\alpha}+\vec{D}\cdot\vec{\tau}^{\dot{\alpha}}_{\dot{\beta}}\bar{\omega}^{\dot{\beta}\alpha}\omega_{\dot{\alpha}\alpha}\Big] \\ & +N_c\mathrm{Tr}\int dtA_0\ . \end{aligned} \tag{5}$$

As we can see, (5) describes a quantum mechanical system with $U(k)$ symmetry where k is the baryon number and the trace is taken over $U(k)$ adjoint representation. Here $M=1,2,3,4$ and $\lambda=g_{YM}^2N_c$ is the 't Hooft coupling. The unique dimension-ful constant M_{KK} is defined as (4). $\vec{D}$ and A_0 are auxiliary fields while the fields X^M and ω are the dynamical fields. ζ is defined as $\zeta=U_{Q_0}^3/U_{KK}^3$. Note that all the fields are bosonic and this matrix model describes the k-baryon system in D0-D4/D8 system according to the holographic principle.

The matrix model (5) is a deformed matrix model of [24]. Without smeared D0-branes, i.e. setting $\zeta=0$, (5) returns back to the matrix model in [24]. Therefore the matrix model (5) is also a deformed ADHM matrix model as claimed in [24]. By integrating out of the auxiliary field $\vec{D}$, it yields a potential of a commutator term as $(\mathrm{Tr}\,[X,X])^2$. So our matrix model looks also close to the BFSS Matrix theory [28] (which is understood as an effective description of M-theory) or the IKKT matrix model [30] if the last term and the mass term are absent. Note that the quadratic term of X^4 and ω would be negative if $\zeta>2$ which corresponds to a system with imaginary mass. It may be understood as the constraint for the stable states of baryons in D0-D4 system i.e. if $\zeta>2$, baryons may not be stable in this system. This viewpoint is exactly the same as [27] by using the instanton views for baryons. Since we will use our matrix model to describe baryons in D0-D4/D8 system, thus only $\zeta<2$ is considered here for stable baryons.

As low energy effective theory, all the fields in our matrix model (5) is similar to the matrix model of [24]. Therefore we employ the symmetry for our matrix model from [24]. And the representation of the fields is summarized in Table 1. The covariant derivative is defined as

$$\begin{aligned} D_0X^M &= \partial_0X^M-i\left[A_0,X^M\right], \\ D_0\omega=\partial_0\omega-iA_0\omega \quad &; \quad D_0\bar{\omega}=\partial_0\bar{\omega}+iA_0\bar{\omega}. \end{aligned} \tag{6}$$

So the matrix model (5) is a quantum mechanical system with the following symmetry

$$U(k)\times SU(N_f)\times SO(3)\,,$$

Fields	index	$U(k)$	$SU(N_f)$	$SU(2)\times SU(2)$
X^M	$M=1,2,3,4$	adj	1	(2,2)
$\omega_{i\dot{\alpha}}$	$\dot{\alpha}=1,2;i=1,2...N_f$	adj	fund	(1,2)
A_0		adj	1	(1,1)
D_s	$s=1,2,3$	1	1	(1,3)

Table 1: Fields in the matrix model

N_f is the number of the flavors in QCD. The first symmetry $U(k)$ is a local symmetry while the symmetry $SU(N_f)\times SO(3)$ is a global symmetry. $SO(3)$ could be interpreted as the rotation symmetry in our space where the baryons live. If we embed the rotation symmetry as

$$SO(3)\subset SO(4)\simeq SU(2)\times SU(2),$$

then the symmetry of the action would be easier to understand. And the additional dimension corresponds to the holographic dimension. By the mass deformation, the $SO(4)$ symmetry breaks down to the $SO(3)$ symmetry.

3.2 Derivation from Holography

In D0-D4/D8 system, our concern is the D4'-branes wrapped on S^4 in the background (1), whose low energy effective theory is described by the matrix model (5). In AdS/CFT duality, such a D4'-brane is named as the "baryon vertex" [15], which is responsible for creating or annihilating baryonic states in the dual field theory.

Since the D4'-branes live inside the flavor branes, the action of the D4'-brane is related to the background geometry, the R-R flux and also affected by the presence of the flavor branes. So in the low energy effective theory of baryonic D4'-branes, the strings connecting the baryon vertices and the flavor branes provide the field ω in the bi-fundamental representation. The location of the D4'-branes in the transverse directions is specified by the diagonal eigenvalues of the field X^M in the adjoint representation. Thus these diagonal eigenvalues represent the locations of the baryons in our real 3-dimensional space. Due to the curved geometry and the flux, only the bosonic fields are kept here since the deformation between our matrix model and ADHM matrix model breaks the supersymmetry explicitly. So we will not care about the fermion part in our theory.

As in [24], we assume that the S^4 dependence can be trivially reduced although the D4'-branes are inside the flavor branes and wrapped on S^4, that means a dimensional reduction with no dependence along S^4. Therefore the derived action is in the time dimension only i.e. depended on time only. So next, we will derive the low energy effective action of k brane vertices system by using standard technique in string theory.

3.2.1 Derivation from DBI part

Let us start from the action for a single D4'-brane which is

$$\begin{aligned} S_{D4'} &= S_{DBI}+S_{CS}\ , \\ S_{DBI} &= -T_{D4}\int d^5\xi e^{-\phi}\sqrt{-\det\left(G_{MN}+2\pi\alpha' F_{MN}\right)}, \\ S_{CS} &= \frac{1}{2\pi}\int C_3\wedge F_2. \end{aligned} \tag{7}$$

Here we have used ξ to represent the coordinates on baryon vertex. The convenient coordinates are used here as in [11, 25]

$$
\begin{aligned}
y &= r\cos\theta \quad ; \quad Z = r\sin\theta, \\
U^3 &= U_{KK}^3 + U_{KK} r^2 \quad ; \quad \theta = \frac{3}{2} \frac{U_{KK}^{1/2}}{R^{3/2} H_0^{1/2} (U_{KK})} \tau,
\end{aligned} \tag{8}
$$

where the flavor D8-brane is located at $y = 0$. We consider a stable D4'-brane situated at $r = 0$ wrapped on S^4, in this case the DBI action is

$$
S_{DBI} = -\frac{T_{D4'}\omega_4}{g_s} \int dt H_0^{1/4} U R^3 (R/U)^{3/4} \sqrt{-G_{00}}, \tag{9}
$$

where the induced metric is

$$
G_{00} = \left(\frac{U}{R}\right)^{3/2} H_0^{1/2} \left[-1 + \left(\partial_0 X^i\right)^2\right] + \frac{4}{9} \frac{U_{KK}}{U} \left(\frac{R}{U}\right)^{3/2} H_0^{1/2} \left(\partial_0 Z\right)^2. \tag{10}
$$

We have induced the metric on the worldvolume of D4'-branes to the low energy effective theory and the index of X^i is $i = 1, 2, 3$, i.e. runs for 3-dimensional space, so we obtain

$$
S_{DBI} = -\frac{T_{D4'}\omega_4}{g_s} \int dt H_0^{1/2} U R^3 \sqrt{1 - \left(\partial_0 X^i\right)^2 - \frac{4}{9} \frac{U_{KK}}{U} \left(\frac{R}{U}\right)^3 \left(\partial_0 Z\right)^2}. \tag{11}
$$

Since we keep k baryons at short distance and only the low energy effective theory is considered here, so we just need to expand (11) for small Z and small X and define

$$
\begin{aligned}
X^4 &= \frac{2}{3} \left(\frac{R}{U_{KK}}\right)^{3/2} Z \ , \\
\zeta &= \frac{U_{Q_0}^3}{U_{KK}^3} \ ,
\end{aligned} \tag{12}
$$

yields a quadratic action

$$
S_{DBI} = \frac{\lambda N_c M_{KK}}{27\pi} (1+\zeta)^{3/2} \int dt \left[-1 + \frac{1}{2} \left(\partial_0 X^M\right)^2 - \frac{1}{6} (\zeta - 2) M_{KK}^2 \left(X^4\right)^2\right]. \tag{13}
$$

The high orders of X, Z and their derivatives have been dropped off.

The kinetic term and mass term for X in matrix action (5) is given by (13). However this leaves the mass term for ω. We have assumed that the mass term for ω is

$$
-\frac{1}{12} (\zeta - 2) M_{KK}^2 \bar{\omega}_i^{\dot{\alpha}} \omega_{i\dot{\alpha}}. \tag{14}
$$

This (14) is a natural guess from a comparison with [24]. Since our matrix model is the deformation from the model in [24], we simply set the mass term of ω as $1/4$ times of the mass term of X^4 which is an assumption from [24].

3.2.2 Commutator terms

We will compute the commutator terms in the matrix action (5). First we expand the generic Dirac-Born-Infield (DBI) action of a Dp-brane (7) to the quadratic order

$$S_{DBI} \backsimeq (2\pi\alpha')^2 \frac{1}{4} T_{Dp} \int d^5\xi e^{-\phi} \sqrt{-\det G_{MN}} G_{MN} G_{PQ} F^{MP} F^{NQ} \ . \tag{15}$$

The relevant relations from T-duality here is $(2\pi\alpha') A_M = X^N G_{NM}$. We therefore have

$$\begin{aligned}
(2\pi\alpha')^2 G_{MN} G_{PQ} F^{MP} F^{NQ} &= 2G^{00} G_{ij} D_0 X^i D_0 X^j + 2G^{00} G_{zz} D_0 Z D_0 Z \\
&\quad - \frac{1}{(2\pi\alpha')^2} \left[X^i, X^j\right] \left[X^k, X^l\right] G_{ik} G_{jl} \\
&\quad - \frac{2}{(2\pi\alpha')^2} \left[X^i, Z\right] \left[X^j, Z\right] G_{ij} G_{zz} \\
&= -2 \left(D_0 X^M\right)^2 - \frac{4}{3^6\pi^2} (1+\zeta)^4 \lambda^2 M_{KK}^4 \left[X^M, X^N\right]^2 \ .
\end{aligned} \tag{16}$$

We have used the value of the metric at $U = U_{KK}$. The commutator term can be rewritten by an auxiliary field $\overrightarrow{D}$ if we consider the following action

$$S = c \int dt \mathrm{Tr} \left[2 (2\pi\alpha')^2 \vec{D}^2 + \vec{D} \cdot \vec{\tau}^{\dot{\alpha}}_{\ \dot{\beta}} \bar{a}'^{\dot{\beta}\alpha} a'_{\alpha\dot{\alpha}}\right] \ . \tag{17}$$

Integrating out the field $\vec{D}$, yields

$$S = c \int dt \mathrm{Tr} \left[\frac{1}{16\pi^2\alpha'^2} \left[a'_m, a'_n\right]^2\right] \ . \tag{18}$$

Comparing (18) with (13), we have

$$S = \frac{\lambda N_c M_{KK}}{54\pi} (1+\zeta)^{3/2} \int dt \left[\frac{3^6\pi^2}{4\lambda^2 M_{KK}^4} \frac{1}{(1+\zeta)^4} \left(\vec{D}\right)^2 + \vec{D} \cdot \vec{\tau}^{\dot{\alpha}}_{\ \dot{\beta}} \bar{X}^{\beta\alpha} X_{\alpha\dot{\alpha}}\right] \ , \tag{19}$$

3.2.3 Chern-Simons term

Finally, let us consider the Chern-Simons part of the Dp-brane action. Since $f_4 = dC_3$ is given in (2), thus the Chern-Simons term in (7) can be rewritten as

$$\begin{aligned}
S_{CS} &= \frac{1}{2\pi} \int C_3 \wedge F_2 \\
&= \frac{1}{2\pi} \frac{1}{2 \cdot 3!} \mathrm{Tr} \int d^5\xi \epsilon^{\mu_1\mu_2\mu_3\alpha\beta} C_{\mu_1\mu_2\mu_3} F_{\alpha\beta} \\
&= N_c \mathrm{Tr} \int dt A_0,
\end{aligned} \tag{20}$$

(20) is nothing but the last term in matrix action (13).

4 Baryon spectrum

In this section, we will use our matrix model (5) to study baryon spectrum of D0-D4/D8 system for $k=1$ case, i.e the single baryon case. So first let us compute the Hamiltonian for a single baryon $k=1$ with general N_f first. Since for $k=1$ the field X is a number, thus all commutators with X could be dropped. This leaves the terms of ω.

$$\begin{aligned} S_{\vec{D}} &= \frac{\lambda N_c M_{KK}}{54\pi}(1+\zeta)^{3/2}\int dt\left[\frac{3^6\pi^2}{4\lambda^2 M_{KK}^4}\frac{1}{(1+\zeta)^4}\left(\vec{D}\right)^2+\vec{D}\cdot\vec{\tau}^{\dot{\alpha}}_{\dot{\beta}}\bar{\omega}^{\dot{\beta}\alpha}\omega_{\dot{\alpha}\alpha}\right] \\ &= -\frac{\lambda N_c M_{KK}}{54\pi}(1+\zeta)^{3/2}\int dt\frac{\lambda^2 M_{KK}^4}{3^6\pi^2}(1+\zeta)^4\left[4\omega_1^i\omega_2^{i*}\omega_2^j\omega_1^{j*}+\left(\omega_1^i\omega_1^{i*}\right)^2\right. \\ &\quad \left.+\left(\omega_2^i\omega_2^{i*}\right)^2-2\omega_1^i\omega_1^{i*}\omega_2^i\omega_2^{i*}\right]. \end{aligned} \tag{21}$$

The so-called ADHM potential is given by (21). Basically we should solve it for the construction of instantons in the ADHM formalism, however it is equivalent to minimize the ADHM potential here. And since there is no dynamics for A_0 in our matrix action (5), thus we can integrate it out and the terms including A_0 become

$$\begin{aligned} S_{A_0} &= \frac{\lambda N_c M_{KK}}{54\pi}(1+\zeta)^{3/2} \\ &\quad\times\int dt\left[iA_0\bar{\omega}_i^{\dot{\alpha}}\partial_0\omega_{\dot{\alpha}i}-i\partial_0\bar{\omega}_i^{\dot{\alpha}}A_0\omega_{\dot{\alpha}i}+(A_0)^2\bar{\omega}_i^{\dot{\alpha}}\omega_{\dot{\alpha}i}+\frac{54\pi}{\lambda M_{KK}}\frac{1}{(1+\zeta)^{3/2}}A_0\right]. \end{aligned} \tag{22}$$

So the equation of motion for A_0 is

$$i\bar{\omega}_i^{\dot{\alpha}}\partial_0\omega_{\dot{\alpha}i}-i\partial_0\bar{\omega}_i^{\dot{\alpha}}\omega_{\dot{\alpha}i}+2A_0\bar{\omega}_i^{\dot{\alpha}}\omega_{\dot{\alpha}i}+\frac{54\pi}{\lambda M_{KK}}\frac{1}{(1+\zeta)^{3/2}}=0. \tag{23}$$

By inserting (23) to (22) we obtain

$$S_{A_0}=\frac{\lambda N_c M_{KK}}{54\pi}(1+\zeta)^{3/2}\int dt\left[-\frac{1}{4\bar{\omega}_i^{\dot{\alpha}}\omega_{\dot{\alpha}i}}\left(i\bar{\omega}_i^{\dot{\alpha}}\partial_0\omega_{\dot{\alpha}i}-i\partial_0\bar{\omega}_i^{\dot{\alpha}}\omega_{\dot{\alpha}i}+\frac{54\pi}{\lambda M_{KK}}\frac{1}{(1+\zeta)^{3/2}}\right)^2\right]. \tag{24}$$

Then we use the definition of the momentum conjugate to the field ω

$$\begin{aligned} P_i^{\dot{\alpha}} &= \frac{\partial S}{\partial\left(\partial_0\omega_{\dot{\alpha}}^i\right)} \\ &= \frac{\lambda N_c M_{KK}}{54\pi}(1+\zeta)^{3/2}\left[\partial_0\bar{\omega}_i^{\dot{\alpha}}-\frac{1}{2\bar{\omega}_j^{\dot{\gamma}}\omega_{\dot{\gamma}j}}\left(i\bar{\omega}_k^{\dot{\beta}}\partial_0\omega_{\dot{\beta}k}-i\partial_0\bar{\omega}_k^{\dot{\beta}}\omega_{\dot{\beta}k}+\frac{54\pi}{\lambda M_{KK}}\frac{1}{(1+\zeta)^{3/2}}\right)i\bar{\omega}_i^{\dot{\alpha}}\right]. \end{aligned} \tag{25}$$

Therefore we obtain the Hamiltonian

$$
\begin{aligned}
H &= P_i^{\dot{\alpha}}\partial_0\omega_i^{\dot{\alpha}} + \bar{P}_i^{\dot{\alpha}}\partial_0\bar{\omega}_i^{\dot{\alpha}} - L \\
&= \frac{\lambda N_c M_{KK}}{54\pi}(1+\zeta)^{3/2}\left[\partial_0\bar{\omega}_i^{\dot{\alpha}}\partial_0\omega_i^{\dot{\alpha}} + \frac{1}{6}\left(1-\frac{1}{2}\zeta\right)M_{KK}^2\bar{\omega}_i^{\dot{\alpha}}\omega_{i\dot{\alpha}}\right. \\
&\quad + \frac{\lambda^2 M_{KK}^4}{3^6\pi^2}(1+\zeta)^4\left(4\omega_1^i\omega_2^{i*}\omega_2^j\omega_1^{j*} + \left(\omega_1^i\omega_1^{i*}\right)^2 + \left(\omega_2^i\omega_2^{i*}\right)^2 - 2\omega_1^i\omega_1^{i*}\omega_2^i\omega_2^{i*}\right) \\
&\quad \left. + \frac{1}{4\bar{\omega}_i^{\dot{\alpha}}\omega_{\dot{\alpha}i}}\left(\left(\frac{54\pi}{\lambda M_{KK}}\right)^2\frac{1}{(1+\zeta)^3} + \left(\bar{\omega}_i^{\dot{\alpha}}\partial_0\omega_{\dot{\alpha}i} - \partial_0\bar{\omega}_i^{\dot{\alpha}}\omega_{\dot{\alpha}i}\right)^2\right)\right]. \qquad (26)
\end{aligned}
$$

4.1 Single flavor

For single flavor case, i.e. $N_f = 1$, we use the following ansatz for ω

$$\omega_{\dot{\alpha}} = \rho_{\dot{\alpha}};\ \bar{\omega}_{\dot{\alpha}} = \rho_{\dot{\alpha}}^*. \qquad (27)$$

Then the Hamiltonian (26) is

$$
\begin{aligned}
H &= \frac{\lambda N_c M_{KK}}{54\pi}(1+\zeta)^{3/2} \\
&\times\left[\frac{1}{2\rho^2}\left(\frac{27\pi}{\lambda M_{KK}}\right)^2\frac{1}{(1+\zeta)^3} + \frac{1}{3}\left(1-\frac{1}{2}\zeta\right)M_{KK}^2\rho^2 + \frac{4\lambda^2 M_{KK}^4}{3^6\pi^2}(1+\zeta)^4\rho^4\right], \qquad (28)
\end{aligned}
$$

where we have used the definition $2\rho^2 = \rho_1\rho_1^* + \rho_2\rho_2^*$. Let us analyze the terms in (28). In the soliton picture, the first term can be interpreted as a self-repulsion of the instanton which is induced by the Chern-Simons term with the path-integration of A_0. The second term comes from the mass term of the matrix model and the curved background geometry while the third term comes from the path-integration over auxiliary field $\vec{D}$, which corresponds to the ADHM potential. Thus all the terms are physical and modified by the appearance of smeared D0-branes.

It is easy to find that the Hamiltonian could be minimized by a nonzero ρ. In the large λ limit, we can reduce the Hamiltonian (28) to a linear formula by putting $\rho = x\lambda^\alpha M_{KK}^{-1}$. Here both x and α are constants. And similar as done in [24], every term in (28) scales with large λ limit. As a result, the second term in (28) can be negligible, then the value for ρ to minimize this Hamiltonian is computed as

$$\rho = 2^{-2/3}3^2\pi^{2/3}\lambda^{-2/3}M_{KK}^{-1}(1+\zeta)^{-7/6}, \qquad (29)$$

and

$$H_{min} = 2^{-5/3}\pi^{-1/3}\lambda^{1/3}N_c M_{KK}(1+\zeta)^{5/6}. \qquad (30)$$

4.2 Two flavors

In this subsection, we need to focus on a more realistic case which has two flavors. In order to eliminate the contribution from the ADHM potential, we first have to satisfy the ADHM constraints $\vec{\tau}^{\dot{\alpha}}_{\dot{\beta}}\bar{\omega}_i^{\dot{\beta}}\omega_{\dot{\alpha}i} = 0$, or equivalently, use the following ansatz

$$\sum_{i=1}^{N_f}\omega^i_{\dot\alpha=1}\omega^{i*}_{\dot\alpha=2}=\sum_{i=1}^{N_f}\omega^i_{\dot\alpha=2}\omega^{i*}_{\dot\alpha=1}=0;\ \sum_{i=1}^{N_f}\left|\omega^i_{\dot\alpha=1}\right|^2=\sum_{i=1}^{N_f}\left|\omega^i_{\dot\alpha=2}\right|^2\ . \tag{31}$$

The ADHM potential disappears if this condition is satisfied. And without loss of generality, this can be achieved by using the following choice

$$\omega^{i=1}_{\dot\alpha}=\begin{pmatrix}\rho\\0\end{pmatrix}_{\dot\alpha};\ \omega^{i=2}_{\dot\alpha}=\begin{pmatrix}0\\\rho\end{pmatrix}_{\dot\alpha}. \tag{32}$$

After including X^4-dependence, the Hamiltonian is

$$\begin{aligned}H &= \frac{\lambda N_c M_{KK}}{54\pi}(1+\zeta)^{3/2}\left[\frac{1}{2\rho^2}\left(\frac{27\pi}{\lambda M_{KK}}\right)^2\frac{1}{(1+\zeta)^3}+\frac{1}{3}\left(1-\frac{1}{2}\zeta\right)M_{KK}^2\rho^2\right.\\ &\left.+\frac{2}{3}\left(1-\frac{1}{2}\zeta\right)M_{KK}^2\left(X^4\right)^2\right]. \end{aligned}\tag{33}$$

This can be minimized at

$$\rho=2^{-1/4}3^{7/4}\pi^{1/2}\lambda^{-1/2}\left(1-\frac{1}{2}\zeta\right)^{-1/4}(1+\zeta)^{-3/4}M_{KK}^{-1}\ . \tag{34}$$

Then the minimum value of the Hamiltonian is

$$H_{min}=\frac{N_c M_{KK}}{\sqrt{6}}\sqrt{1-\frac{1}{2}\zeta}\ . \tag{35}$$

In the soliton approach [18, 27], the value of ρ is the size of instantons. Obviously in our theory, we find that all the values are modified by ζ which is related to the appearance of smeared D0-branes and (35) would be totally complex if $\zeta>2$.

4.3 Quantization

Since we have obtained the results on the vacuum of the matrix model, we can then quantize the Hamiltonian for the $k=1,\ N_f=2$ circumstance and the results should correspond to the baryon spectrum. We first rewrite the Hamiltonian used in [18] since we are going to use the same tricks, so we have

$$H=\frac{\lambda N_c M_{KK}}{54\pi}\left[\frac{2}{5\rho^2}\left(\frac{27\pi}{\lambda M_{KK}}\right)^2+\frac{1}{3}M_{KK}^2\rho^2+\frac{2}{3}M_{KK}^2\left(X^4\right)^2\right]\ . \tag{36}$$

By comparing (36) with (33), the energy spectrum in our system can be obtained easily by replacing the Q and ω_ρ,ω_Z used in [18] as

$$Q\to\frac{5}{4}Q(1+\zeta)^{-3/2};\ (\omega_\rho\text{ or }\omega_Z)\to(1+\zeta)^{3/4}\left(1-\frac{1}{2}\zeta\right)^{1/2}(\omega_\rho\text{ or }\omega_Z)\ . \tag{37}$$

Then the mass formula for the baryon excitation is

$$M=M_0+(1+\zeta)^{3/4}\left(1-\frac{1}{2}\zeta\right)^{1/2}\left[\sqrt{\frac{(l+1)^2}{6}+\frac{N_c^2}{6}(1+\zeta)^{-3/2}}+\frac{2(n_\rho+n_z)+2}{\sqrt{6}}\right], \tag{38}$$

where $M_0=\frac{\lambda N_c M_{KK}}{27\pi}$. In order to fit the data from experimental values, we set $\zeta=1.9303$ with $N_c=3$ for real QCD and set $M_{KK}=945\text{MeV}$ to fit the mass of ρ meson. The baryon mass spectrum from our matrix model (38) is listed in Table 2 which is more close to the experimental data . As a comparison, we also list experimental data in Table 3 and the superscripts $\pm$ represent the parity. The subscript $*$ here is used to indicate that evidence for the existence of the baryonic states is poor.

Such a baryon spectrum has already been obtained in [18]. However their original results are much larger than the experimental data if fitting the experiment data by setting $M_{KK}=945\text{MeV}$ (mass of ρ meson) with $N_c=3$. And in [24], the baryon spectrum would be still larger than the experimental data if fitting the experiment with the same value for M_{KK} and N_c. Most likely, the reason is that Sakai-Sugimoto model describes the QCD with large N_c limit by holography, but the real QCD is a theory with $N_c=3$. Therefore, in our D0-D4/D8 system, we suggest to give an effective description for $N_c=3$ QCD by adjusting the number density of D0-branes i.e. the parameter ζ in our system. Note that our result (38) does not make sense if $\zeta>2$ since the mass spectrum would be totally imaginary. As mentioned, the stable baryonic state may not exist if $\zeta>2$ in D0-D4/D8 system, which would be quite different from the original Sakai-Sugimoto model.

(n_ρ,n_z)	$(0,0)$	$(1,0)$	$(0,1)$	$(1,1)$	$(2,0)/(0,2)$	$(2,1)/(0,3)$	$(1,2)/(3,0)$
$N\,(l=1)$	945^+	1268^+	1268^-	1590^-	$1590^+,1590^+$	$1913^-,1913^-$	$1913^+,1913^+$
$\Delta\,(l=3)$	1237^+	1560^+	1560^-	1882^-	$1882^+,1882^+$	$2205^-,2205^-$	$2205^+,2205^+$

Table 2: Baryon spectrum of mass from (38)

(n_ρ,n_z)	$(0,0)$	$(1,0)$	$(0,1)$	$(1,1)$	$(2,0)/(0,2)$	$(2,1)/(0,3)$	$(1,2)/(3,0)$
$N\,(l=1)$	940^+	1440	1535^-	1655^-	$1710^+,?$	$2090^-_*,?$	$2100^+_*,?$
$\Delta\,(l=3)$	1232^+	1600^+	1700^-	1940^-	$1920^+,?$	$?,?$	$?,?$

Table 3: Experimental data of baryon mass for various baryonic states

5 Two body baryon interaction

In this section we will consider the interaction between two baryons for two-flavor case since it would be more realistic, thus $N_f,\ k=2$. For the two-flavor case in the matrix model (5), integrating out of the $U(k)$-adjoint field D_{AB} gives the vacuum configuration, which is just the ADHM constraints

$$\vec{\tau}^{\dot{\alpha}}_{\dot{\beta}}\left(\bar{X}^{\dot{\beta}\alpha}X_{\dot{\alpha}\alpha}+\bar{\omega}^{\dot{\beta}\alpha}\omega_{\dot{\alpha}\alpha}\right)_{BA}=0\ , \tag{39}$$

where the indices for baryon are $A,B=1,2...k$. Since our matrix model is also a deformed ADHM matrix model, we will also use the ADHM data for our model as done in [24]

$$\begin{aligned} X^M &= \tau^3\frac{r_M}{2}+\tau^1 Y_M\ , \\ \omega^{A=1}_{\dot{\alpha}i}=U^{A=1}_{\dot{\alpha}i}\rho_1 \quad &; \quad \omega^{A=2}_{\dot{\alpha}i}=U^{A=2}_{\dot{\alpha}i}\rho_2\ , \end{aligned} \tag{40}$$

where r_M is the inter-baryon distance, U^A is the $SU(2)$ matrices which represents the moduli parameters of each baryon and

$$Y_M = -\frac{\rho_1\rho_2}{4\left(r_L\right)^2}\mathrm{Tr}\left[\bar{\sigma}_M r_N \sigma_N \left(\left(U^1\right)^\dagger U^2 - \left(U^2\right)^\dagger U^1\right)\right] \ , \tag{41}$$

we have used $\sigma_M = (i\vec{\tau}, 1)$ and $\bar{\sigma}_M = (-i\vec{\tau}, 1)$. Note that the ADHM data are just the solution for two Yang-Mills instantons which has been explicitly used in the approach of soliton [22]. So after the quantization, their degrees of freedom become the spin and the isospin of each baryon, which are nothing but the gauge rotations of the flavor gauge group. Here we use real unit vectors a_M^A to write them as

$$U^A = a_4^A + i a_i^A \tau^i \ , \tag{42}$$

with $\left(a_4^A\right)^2 + \left(a_i^A\right)^2 = 1$. Then we can obtain some useful expression listed as follows which we are going to use

$$\begin{aligned}
r_M Y_M &= 0 \ , \\
Y_M Y_M &= -\frac{\rho_1^2\rho_2^2}{8\left(r_M\right)^2}\mathrm{Tr}\left[\left(\left(U^1\right)^\dagger U^2 - \left(U^2\right)^\dagger U^1\right)^2\right] = -\frac{\rho_1^2\rho_2^2}{4\left(r_M\right)^2}\left[1 - \left(a_M^1 a_M^2\right)^2\right] \ , \\
\mathrm{Tr}\left[\left(U^1\right)^\dagger U^2\right] &= \mathrm{Tr}\left[\left(U^2\right)^\dagger U^1\right] = 2 a_M^1 a_M^2 \ .
\end{aligned} \tag{43}$$

Then, we shall compute the two-baryon interaction potential. First we write down the terms in the action (5) with ADHM data

$$\mathrm{Tr}\left(D_0 X^M\right)^2 = 2\left[\left(A_0^1\right)^2 r_M^2 + \left(A_0^2\right)^2\left(r_M^2 + 4Y_M^2\right) + 4\left(A_0^3\right)^2 Y_M^2\right] - 8A_0^1 A_0^3 r_M Y_M \ . \tag{44}$$

Note that the last term in (44) vanishes once (43) is added. Then, for the kinetic term of ω we have

$$\begin{aligned}
\mathrm{Tr}\left(D_0 \bar{\omega}_i^{\dot{\alpha}} D_0 \omega_{i\dot{\alpha}}\right) &= 2\left(\rho_1^2 + \rho_2^2\right)\left[\left(A_0^0\right)^2 + \left(A_0^1\right)^2 + \left(A_0^2\right)^2 + \left(A_0^3\right)^2\right] \\
&\quad + 4\rho_1\rho_2 A_0^0 A_0^1 \mathrm{Tr}\left[\left(U^1\right)^\dagger U^2\right] + 4\left(\rho_1^2 - \rho_2^2\right) A_0^0 A_0^3 \ .
\end{aligned} \tag{45}$$

We can minimize it with $A_0^2 = 0$ since the component A_0^2 appears only as $\left(A_0^2\right)^2$. Thus the resulting baryon interaction potential V from the kinetic term plus Chern-Simons term can be evaluated by $\int dtV = -S_{\mathrm{on-shell}}$, which is

$$\begin{aligned}
S_{kinetic}^{\mathrm{on-shell}} &= \frac{\lambda N_c M_{KK}}{54\pi}\left(1+\zeta\right)^{3/2}\mathrm{Tr}\int dt\left[\left(D_0 X^M\right)^2 + D_0\bar{\omega}_i^{\dot{\alpha}} D_0 \omega_{i\dot{\alpha}}\right] + N_c \mathrm{Tr}\int dt A_0 \\
&= \frac{\lambda N_c M_{KK}}{54\pi}\left(1+\zeta\right)^{3/2}\int dt\Big[2\left(A_0^1\right)^2 r_M^2 + 8\left(A_0^3\right)^2 Y_M^2 \\
&\quad + 2\left(\rho_1^2+\rho_2^2\right)\left(\left(A_0^0\right)^2 + \left(A_0^1\right)^2 + \left(A_0^3\right)^2\right) \\
&\quad + 4\rho_1\rho_2 A_0^0 A_0^1 \mathrm{Tr}\left(\left(U^1\right)^\dagger U^2\right) + 4\left(\rho_1^2-\rho_2^2\right) A_0^0 A_0^3 + \frac{108\pi}{\lambda M_{KK}}\left(1+\zeta\right)^{-3/2} A_0\Big] \ .
\end{aligned} \tag{46}$$

Since the action (46) does not depend on the derivative of A_0, we can integrate them out straightforwardly, therefore the resulting baryon potential is

$$V=\frac{27\pi N_c}{\lambda M_{KK}}\frac{1}{(1+\zeta)^{3/2}}\frac{1}{\rho_1^2\rho_2^2}\frac{\left(\rho_1^2+\rho_2^2+r_M^2\right)\left(u\rho_1^2\rho_2^2-4\left(\rho_1^2+\rho_2^2\right)r_M^2\right)}{\left[u\left(\rho_1^4-(2+u)\rho_1^2\rho_2^2+\rho_2^4\right)+5u\left(\rho_1^2+\rho_2^2\right)r_M^2-16\left(r_M^2\right)^2\right]}\ , \tag{47}$$

where we have defined $u=\left(\mathrm{Tr}\left[\left(U^1\right)^\dagger U^2\right]\right)^2-4$. And there is another part of baryon interaction potential, which comes from the mass term of X^4 in action (5) in addition to this potential (47). It can be evaluated as

$$\begin{aligned}&\tfrac{\lambda N_c M_{KK}}{54\pi}\left(1+\zeta\right)^{3/2}\tfrac{2}{3}\left(1-\tfrac{1}{2}\zeta\right)M_{KK}^2\mathrm{Tr}\left(X^4\right)^2\\ =\;&\tfrac{\lambda N_c M_{KK}}{81\pi}\left(1+\zeta\right)^{3/2}\left(1-\tfrac{1}{2}\zeta\right)M_{KK}^2\left(\tfrac{r_4^2}{2}+Y_4^2\right)\quad .\end{aligned} \tag{48}$$

This term (48) does not contribute to the two-baryon interaction since the term of r_4^2 is the mass term in the single-baryon Hamiltonian. In fact, the off-diagonal elements of Y_4 contribute to the interaction between the baryons. Then using (41) we have

$$Y_4=-\frac{\rho_1\rho_2}{2r_M^2}r_i\mathrm{Tr}\left(i\tau^iU^{(1)\dagger}U^{(2)}\right)\ , \tag{49}$$

where $i=1,2,3$, so we can rewrite the potential energy (48) as

$$\frac{\lambda N_c M_{KK}}{162\pi}\left(1+\zeta\right)^{3/2}\left(1-\frac{1}{2}\zeta\right)M_{KK}^2\left[r_4^2+\frac{\rho_1^2\rho_2^2}{\left(r_M^2\right)^2}\left(r_i\mathrm{Tr}\left[i\tau^iU^{(1)\dagger}U^{(2)}\right]\right)^2\right]\ . \tag{50}$$

Therefore, putting all together, we obtain the two-body interaction Hamiltonian of baryons which is

$$\begin{aligned}V\;=\;&\frac{27\pi N_c}{\lambda M_{KK}}\frac{1}{(1+\zeta)^{3/2}}\frac{1}{\rho_1^2\rho_2^2}\frac{\left(\rho_1^2+\rho_2^2+r_M^2\right)\left(u\rho_1^2\rho_2^2-4\left(\rho_1^2+\rho_2^2\right)r_M^2\right)}{\left[u\left(\rho_1^4-(2+u)\rho_1^2\rho_2^2+\rho_2^4\right)+5u\left(\rho_1^2+\rho_2^2\right)r_M^2-16\left(r_M^2\right)^2\right]}\\ &+\frac{\lambda N_c M_{KK}}{162\pi}\left(1+\zeta\right)^{3/2}\left(1-\frac{1}{2}\zeta\right)M_{KK}^2\left[\frac{\rho_1^2\rho_2^2}{\left(r_M^2\right)^2}\left(r_i\mathrm{Tr}\left[i\tau^iU^{(1)\dagger}U^{(2)}\right]\right)^2\right]\\ &-\frac{27\pi N_c}{4\lambda M_{KK}}\frac{1}{(1+\zeta)^{3/2}}\left(\frac{1}{\rho_1^2}+\frac{1}{\rho_2^2}\right)\ .\end{aligned} \tag{51}$$

If we assume that the size ρ of the baryons or nucleons is much small, then we can expand (51) for $r_M\gg\rho$ and obtain a leading term which is

$$\begin{aligned}V\;=\;&\frac{27\pi N_c}{64\lambda M_{KK}}\frac{1}{(1+\zeta)^{3/2}}\frac{1}{r_M^2}\left[32+6u+(5u+16)\left(\frac{\rho_1^2}{\rho_1^2}+\frac{\rho_2^2}{\rho_1^2}\right)\right]\\ &+\frac{\lambda N_c M_{KK}^3}{162\pi}\left(1+\zeta\right)^{3/2}\left(1-\frac{1}{2}\zeta\right)\left[\frac{\rho_1^2\rho_2^2}{\left(r_M^2\right)^2}\left(r_i\mathrm{Tr}\left[i\tau^iU^{(1)\dagger}U^{(2)}\right]\right)^2\right]\ ,\end{aligned} \tag{52}$$

again by choosing $\rho_1=\rho_2=\rho$, we have a formula from (52)

$$
\begin{aligned}
V &= \frac{27\pi N_c}{4\lambda M_{KK}}\frac{1}{(1+\zeta)^{3/2}}\frac{1}{r_M^2}\left(\mathrm{Tr}\left[U^{(1)\dagger}U^{(2)}\right]\right)^2 \\
&+\frac{\lambda N_c M_{KK}^3}{162\pi}(1+\zeta)^{3/2}\left(1-\frac{1}{2}\zeta\right)\left[\frac{\rho^4}{\left(r_M^2\right)^2}\left(r_i\mathrm{Tr}\left[i\tau^i U^{(1)\dagger}U^{(2)}\right]\right)^2\right]. \qquad (53)
\end{aligned}
$$

Then, let us compute the vacuum expectation value by using the potential (53). We use $\left(\vec{I}_A,\vec{J}_A,n_\rho,n_z\right)$ to label the states of the two baryons, with $A=1,2$ representing the two baryons. The excited baryon states are labeled by n_ρ and n_Z which are the quantum numbers. And $\vec{I},\vec{J}$ is the isospin (spin) of the baryon state respectively. For nucleons we have $\left(\left|\vec{I}\right|=\left|\vec{J}\right|=\frac{1}{2}\right)$ and the explicit spin/isospin wave functions reads [22, 24]

$$
\frac{1}{\pi}\left(\tau^2 U\right)_{IJ}=\begin{pmatrix} |p\uparrow> & |p\downarrow> \\ |n\uparrow> & |n\downarrow> \end{pmatrix}. \qquad (54)
$$

The baryon potential is evaluated as

$$
\begin{aligned}
\left\langle\left(\mathrm{Tr}\left[U^{(1)\dagger}U^{(2)}\right]\right)^2\right\rangle_{I_1,J_1,I_2,J_2} &= 1+\frac{16}{9}\left(\vec{I}_1\cdot\vec{I}_2\right)\left(\vec{J}_1\cdot\vec{J}_2\right), \\
\left\langle\mathrm{Tr}\left[i\tau^i U^{(1)\dagger}U^{(2)}\right]\mathrm{Tr}\left[i\tau^j U^{(1)\dagger}U^{(2)}\right]\right\rangle_{I_1,J_1,I_2,J_2} &= \delta^{ij}+\frac{16}{9}\vec{I}_1\cdot\vec{I}_2\left(J_1^iJ_2^j+J_2^jJ_1^i-\delta^{ij}\vec{J}_1\cdot\vec{J}_2\right). \qquad (55)
\end{aligned}
$$

By the standard definition of $S_{12}=12\left(\vec{J}_1\cdot\hat{\vec{r}}\right)\left(\vec{J}_2\cdot\hat{\vec{r}}\right)-4\vec{J}_1\cdot\vec{J}_2$ with $\hat{\vec{r}}=\vec{r}/\left|\vec{r}\right|$, we have

$$
\begin{aligned}
V_C^{(0)}(\vec{r}) &= \pi\left[\frac{3^3}{2}+8\left(\vec{I}_1\cdot\vec{I}_2\right)\left(\vec{J}_1\cdot\vec{J}_2\right)\right]\frac{N_c}{\lambda M_{KK}}\frac{1}{(1+\zeta)^{3/2}}\frac{1}{r^2}, \\
V_T^{(0)}(\vec{r}) &= 2\pi\left(\vec{I}_1\cdot\vec{I}_2\right)\frac{N_c}{\lambda M_{KK}}\frac{1}{(1+\zeta)^{3/2}}\frac{1}{r^2}. \qquad (56)
\end{aligned}
$$

For the leading order which comes from the second term in (53), we have[3]

$$
\begin{aligned}
V_C^{(1)}(\vec{r}) &= \left[\frac{1}{81}-\frac{16}{2187}\left(\vec{I}_1\cdot\vec{I}_2\right)\left(\vec{J}_1\cdot\vec{J}_2\right)\right](1+\zeta)^{3/2}\left(1-\frac{1}{2}\zeta\right)\frac{\lambda N_c M_{KK}^3}{\pi}\frac{\rho^4}{r^2}, \\
V_T^{(1)}(\vec{r}) &= \frac{8}{2187}\left(\vec{I}_1\cdot\vec{I}_2\right)(1+\zeta)^{3/2}\left(1-\frac{1}{2}\zeta\right)\frac{\lambda N_c M_{KK}^3}{\pi}\frac{\rho^4}{r^2}. \qquad (57)
\end{aligned}
$$

Thus our total potential up to leading order is

$$
\begin{aligned}
V_C(\vec{r}) &= V_C^{(0)}(\vec{r})+V_C^{(1)}(\vec{r}), \\
V_T(\vec{r}) &= V_T^{(0)}(\vec{r})+V_T^{(1)}(\vec{r}), \\
V_{nuclear} &= V_C(\vec{r})+S_{12}V_T(\vec{r}). \qquad (58)
\end{aligned}
$$

[3]Here the exact result is $\frac{\rho^4\left(r^i\right)^2}{\left(r^Mr^M\right)^2}$, we simply write it as $\frac{\rho^4}{r^2}$.

This is the short-distance two-body force for baryons in D0-D4/D8 system obtained from our matrix model. With $\zeta < 2$ we also find there is a repulsive core of baryons or nucleons in this system. As in [22, 24], the repulsive potential scales as r^{-2} which was treated as a property peculiar to the holographic model . If comparing (53) (56) (57) with the results in [24], we can see that the nuclear force in [24] is just our results with $\zeta = 0$ i.e. no smeared D0-branes. With the appearance of smeared D0-branes, the zeroth-order result is depressed by the number of D0 while the leading order result increases. As mentioned in previous sections, our model for baryons is consistent with $\zeta < 2$ and this is the condition for the existence of stable baryons in our theory. However an attractive force appears from (57) if $\zeta > 2$ which may give an unstable state for baryons in two-body system by this model.

6 Summary and discussion

We have studied the holographic baryons by the matrix model in the D0-D4/D8 system. We start with considering the baryon vertex inside the flavor branes i.e. D4'-branes wrapped on S^4 which is embedded in the D0-D4 background on large N_c limit. And we use the standard technique in string theory to derive our matrix model. By using T-duality and dimensional reduction, we obtain our matrix model (5) with $U(k)$ symmetry, which could also be able to describe multi-baryons. In order to describe stable baryonic states, we find the value of ζ is restricted to $\zeta < 2$. This is exactly the same as the results in [27]. However in [27], baryons are described by using BPST configurations which is similar as [18]. In our paper, as a difference, we start from the baryon vertex, but we come to the same conclusion i.e. stable baryons exist in D0-D4/D8 system only if $\zeta < 2$. So it is worthy to believe such a result is unique for this system.

With this matrix model, we also determine the holographic size and the spectrum of baryon for the case with $k = 1$ and $N_f = 1, 2$. We find the spectrum obtained from our model could be more close to the experimental data by choosing suitable value of ζ. Thus we suggest that it is an effective description of QCD for low energy baryonic states. Again, we have seen that in the two flavor case the spectrum of baryon's mass would be totally imaginary if $\zeta > 2$. It turns out that baryons in this system are not stable if $\zeta > 2$ as mentioned. In two flavor case, our spectrum of baryon's mass (38) could recover the results in [24] if setting $\zeta = 0$ i.e. no smeared D0-branes. However, we can not recover the results (36) from [18], since the first term in (33) is deformed by a factor $4/5$, even if setting $\zeta = 0$. This puzzle was found in [24] first which makes our spectrum (38), from the matrix model of D0-D4/D8 system, a bit different from the results by the instanton viewpoint in [27]. We are less clear about this and would take further study about this in the future work.

And we also compute the two-body short-distance effective potential for baryons in D0-D4/D8 system i.e. $k = 2$ case. It also exhibits a repulsive core and a tensor force as [22, 24], which has been well-known in nuclear physics, but as a difference, it is modified by the appearance of smeared D0-branes. As it is known the nucleons interact each others by interchanging mesons and the effective potential can be derived from the Yukawa coupling. And on the other hand, in [25, 27], the spectrum of mesons, such as ρ mesons, of Sakai-Sugimoto model in D0-D4 background are modified by the appearance of smeared D0-branes. So in our paper, (56) (57) can be interpreted as follows: since the mass of mesons are modified, thus the effective potential from the interaction are also modified by the appearance of smeared D0 branes. This two-body interaction has not been calculated by using the instanton viewpoints in D0-D4/D8 system in [27], so our work would be advancing on this front. Our effective two-body potential (56) (57) is obtained by expanding (51) respected to ρ/r, however it shows that if $\zeta > 2$, the leading order potential would be negative i.e. it is an attractive force. Furthermore, if ζ and λ are large enough, the zero order potential would be depressed very much while the leading order potential becomes a rapid negative increasing, which makes the total potential negative. A negative two-body short-distance effective potential implies there is an attractive force between baryons or nucleons at short distance, so the system would not be stable. From the effective two-body force, it

also and consistently turns out that ζ is restricted even in two-body system if there would be stable baryonic states.

Finally, we would like to give some more comments about this work. Basically, our analysis is done in large N_c limit as in most analysis in gauge-gravity duality, though we have tried to give an effective description of finite N_c theory in this paper. The interaction between baryons would be important for studying on the phase structure of strong-coupling QCD by holography, such as [16, 19]. So a holographic model corresponded to real QCD would be significant, however unfortunately, currently out of reach. Besides, we can see that our two-body effective potential for baryons makes sense at short distance only, and furthermore we still do not know how to introduce a realistic attractive force consistently or describe the long-distance force by considering the baryon vertex directly. These problems may be solved by calculating the interaction between the D4'-branes in curved spacetime, however it would be a huge challenge in string theory. So it should be understood as that there is still a long way to the realistic and strong-coupling QCD from holography.

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